

### Stage: Pictorial Representation

Follow up by relating the activity to the picture of the scale and the cubes on CWB p. 1. This helps students to transit from the concrete experience to the pictorial representation. In this stage, they will see how the pictorial representation can help them to understand the meaning of an equation, which will be introduced in the Abstract stage.

- Refer students to the picture of the scale and cubes on CWB p. 1.
- Have students look at the two blocks of cubes on the left. Relate these back to the blocks of connecting cubes that they had placed on the left pan earlier. Highlight that there are 6 cubes in all on the left side.
- Next, have students look at the block of cubes on the right. Relate it back to the 6 connecting cubes that they had placed on the right pan when they were trying to balance the scale. Reiterate that there are 6 cubes on the right side.
- Lastly, draw students' attention to the scale. Have them see that the scale is balanced, and the total number of cubes on the left of the scale is equal to the number of cubes on the right.

### Stage: Abstract Representation

In this stage, students transit from the pictorial to the abstract representation as they learn to understand the meaning of an equation and how to write one. Having gone through the previous stages, students will see how the equation relates to the pictorial representation. This forms the basic understanding of equations and serves as an anchor for learning algebraic equations.

- Write ' $2 + 4 = 6$ ' on the board.
- Point out to them that  $2 + 4 = 6$  is an equation.
- Go through the definition of an equation in the note box with students.
- Have students see that the equation  $2 + 4 = 6$  shows the relationship between the number of cubes on both sides of the scale shown on CWB p. 1.
- Draw students' attention to the left side of the equation,  $2 + 4$ , which has a total value of 6. Relate this back to the pictorial representation by having students look at the two blocks of cubes on the left pan in the picture. Lead them to see that 2 represents the block of 2 red cubes, 4 represents the block of 4 yellow cubes and there are 6 cubes altogether on the left pan.
- Next, draw students' attention to the right side of the equation, 6. Relate this back to the pictorial representation by having them look at the block of cubes on the right pan in the picture. Guide them to see that 6 represents the block of 6 orange cubes.
- Highlight to students that the left side and the right side of the equation have the same value, 6. Relate this back to the pictorial

representation by having students look at the balanced scale.

- Reiterate to students that in an equation, the values on both sides of the equal sign are the same. In this example, the left side,  $2 + 4$ , has the same value as the right side, 6.
- Extend students' knowledge about equations by introducing them to another type of equation.
- Write ' $4 + x = 10$ ' and ' $3z - 2 = 4$ ' on the board.
- Point out to students that these are also equations. Have them see that unlike the equation  $2 + 4 = 6$  earlier, each of these equations has an unknown number that is represented by a letter.
- Explain to students that such equations are called algebraic equations.
- Reinforce students' understanding of algebraic equations by writing a few other examples of algebraic equations and expressions (without equal signs) on the board. Then, get them to identify which of them are equations.
- Reiterate that an equation has an equal sign, while an expression does not.

### ❖ Blended Learning Program ❖

#### From PRIME Mathematics Interactive Edition:

##### Let's Do (CB p. 8)

Assign the tasks to students as classwork for formative assessment. Use the corresponding lesson notes to identify the objectives of each task and address remediation needs.

##### Exercise 1 (PB p. 7)

Assign the tasks to students as classwork for further formative assessment. Use the corresponding lesson notes to identify the objectives of each task and address remediation needs.

#### From PRIME Mathematics Coursework Book:

##### Coursework Book Practice 1 (CWB p. 2)

Assign all tasks to students as homework. Use the following notes to identify the skills needed for each task and address remediation needs.

### Practice 1 (CWB p. 2)

#### Class practice (For Print-based Program):

Task 1 requires students to identify an algebraic equation. Students are expected to understand what an equation is, and they are required to recognize that mathematical sentences with an equal sign and an unknown number represented by a letter are algebraic equations.

### Remediation

Task 1(a): Point out to students that in  $x + 5$ , there is an unknown number that is represented by the letter  $x$ . However, there is no equal sign. So,  $x + 5$  is not an algebraic equation. It is an algebraic expression.

Task 1(b): Have students see that in  $q - 4 = 8$ , there is an equal sign as well as an unknown number that is represented by the letter  $q$ . Therefore,  $q - 4 = 8$  is an algebraic equation.

Task 1(c): Guide students to see that in  $10 - 6b + 3 = 1$ , there is an equal sign as well as an unknown number that is represented by the letter  $b$ . So,  $10 - 6b + 3 = 1$  is an algebraic equation.

Task 1(d): Have students see that in  $16 + 3 = 19$ , there is no unknown number that is represented by a letter. Even though there is an equal sign,  $16 + 3 = 19$  is not an algebraic equation. It is just an equation.

Task 1(e): Point out to students that in  $23u - 5 + 3$ , there is an unknown number that is represented by the letter  $u$ . However, there is no equal sign. So,  $23u - 5 + 3$  is not an algebraic equation. It is an algebraic expression.

Task 1(f): Guide students to see that in  $10a + 2a = 3$ , there is an equal sign as well as an unknown number that is represented by the letter  $a$ . So,  $10a + 2a = 3$  is an algebraic equation.

### Teaching tips

#### Task 1

- Reiterate to students that an equation has an equal sign, whereas an expression does not.
- Highlight to students the two things that they will have to look out for when identifying an algebraic equation – unknown number that is represented by a letter, and equal sign.

### Independent practice (For Print-based Program):

Task 2 requires students to identify an algebraic equation. In each part, students are given a pair of mathematical sentences. They are expected to identify the algebraic equation from the given pair by circling it. To identify the algebraic equation, students should look out for an unknown number and an equal sign.

Task 3 requires students to differentiate between an algebraic equation and an algebraic expression. They are expected to tick the appropriate column after identifying if the given mathematical sentence is an algebraic equation or an algebraic expression. Students should look out for an equal sign when identifying algebraic equations.

For answers, go to CW Manual p. 96.

### ❖ Blended Learning Program ❖

#### From PRIME Mathematics Interactive Edition:

Let's Learn (CB pp. 9–10)

Go through the teaching examples with students for concept development. Use the detailed lesson plan given in the corresponding lesson notes to carry out the teaching.

### Learn

#### Using the guess and check method to solve algebraic equations (CWB pp. 3–4)

#### Learning Outcome:

- Use the guess and check method to solve an algebraic equation

#### Vocabulary:

- solution
- solve

#### (a)

Stage: Abstract Representation

Students will now proceed to learn how to solve algebraic equations. Here, students are taught one of the methods that they can use – the guess and check method. In this first example, students will learn to solve a simple algebraic equation using this method. This method allows students to develop an understanding of how an unknown value represented by a letter fits in a basic algebraic equation, and how to find its value. It also allows students to develop logical reasoning by deducing whether the unknown value should be greater or smaller than the guessed value.

- Write ' $9 + m = 16$ ' on the board.
- Guide students to see that to find the value of  $m$ , we need to make guesses. Explain to them that this method of finding the value of the unknown number is known as the guess and check method.
- Point out to students that in this method, we usually start by using small numbers first.
- Lead students to start their guess by using  $m = 2$ .
- Write ' $m = 2: 9 + m = 9 + 2$ ' on the board.  
 $= 11$
- Have students see that when  $m = 2$ , we get an answer of 11.
- Remind students that in an equation, the value on both sides of the equal sign are the same. As such, we want the answer to be 16.
- Have students see that since 11 is smaller than 16, the value of  $m$  cannot be 2.
- Guide students to see that in order to get a value greater than 11, our next guess for the value of  $m$  should be greater than 2.
- Get students to try  $m = 8$ .
- Have a student substitute 8 for  $m$  in the expression ' $9 + m$ ' and work out the answer on the board.
- Write ' $m = 8: 9 + m = 9 + 8$ ' on the board.  
 $= 17$

- Have students see that when  $m = 8$ , we get an answer of 17.
- Highlight to them that since 17 is greater than 16, the value of  $m$  cannot be 8.
- Guide students to see that in order to get a value smaller than 17, our next guess for the value of  $m$  should be smaller than 8 but greater than 2.
- Point out to them that since 17 is close to 16, the value of  $m$  must be close to 8.
- Get students to try  $m = 7$ .
- Have another student substitute 7 for  $m$  in the expression and work out the answer on the board.
- Write ' $m = 7: 9 + m = 9 + 7$ ' on the board.  
 $= 16$
- Have students see that when  $m = 7$ , we get an answer of 16. This is the same as the value on the right side of the equal sign. So, 7 is the correct value of the unknown number  $m$ .
- Explain to students that when they have found the value of the unknown number, it means that they have solved the equation.
- Point out to them that the correct value of the unknown number is known as the solution of the equation. In this example,  $m = 7$  is a solution of the equation  $9 + m = 16$ .
- Go through the note box with students and reiterate that to solve an algebraic equation, we find the value of the unknown number in it.

## (b)

Stage: Abstract Representation

This example is an extension of the example in (a). Students will learn to solve algebraic equations that have a number in front of the letter, using the guess and check method. Similar to example (a), this will allow students to practice their logical reasoning by deducing whether the unknown value should be greater or smaller than the guessed value. This is also an anchor for learning how to solve algebraic equations that involve fractions.

- Write 'Solve  $2x + 5 = 15$ .' on the board.
- Remind students that to solve the equation means to find the value of the unknown letter  $x$  that will make the values on both sides of the equation the same.
- Reiterate to students that we can make guesses to find the value of  $x$ .
- Lead students to start their guess by using  $x = 4$ .
- Get a student to substitute 4 for  $x$  in the expression  $2x + 5$  and work out the answer on the board.
- Write ' $x = 4: 2x + 5 = 2 \times 4 + 5$ ' on the board.  
 $= 8 + 5$   
 $= 13$
- Have students see that when  $x = 4$ , we get an answer of 13.
- Point out to them that we want to get an answer of 15. So, the guess of  $x = 4$  is incorrect.
- Guide students in improving on their next guess with what they have learned from the incorrect guess.

- Lead them to see that since 13 is smaller than 15, the value of  $x$  should be greater than 4. This will enable us to get a value that is greater than 13 when we substitute the number for  $x$  in the expression.
- Have students also see that since 13 is close to 15, the value of  $x$  must be close to 4.
- Get students to try  $x = 5$ .
- Have another student substitute 5 for  $x$  in the expression and work out the answer on the board.
- Write ' $x = 5: 2x + 5 = 2 \times 5 + 5$ ' on the board.  
 $= 10 + 5$   
 $= 15$
- Have students see that when  $x = 5$ , we get an answer of 15. This is the same as the value on the right side of the equal sign. So, we have solved the equation and the guess of  $x = 5$  is correct.
- Conclude that  $x = 5$  is a solution of the equation  $2x + 5 = 15$ .

## (c)

Stage: Abstract Representation

At this point, students should be able to solve simple algebraic equations and those algebraic equations that have a number in front of the letter. In this example, students continue to use the guess and check method to solve algebraic equations that involve fractions. As solving this type of algebraic equations involves multiplying a fraction and a whole number, it is important that students have mastered that concept well. After going through these examples, students will see that it is inefficient to solve algebraic equations using the guess and check method. They will learn that there is a more efficient method of solving algebraic equations, which will be covered in the next lesson.

- Write 'Solve  $\frac{1}{3}n - 9 = 0$ .' on the board.
- Reiterate to students that to solve the equation, they need to find the value of  $n$  that will make the values on both sides of the equation the same.
- Have students see that we are going to use the guess and check method to solve the equation.
- Highlight to students that in this equation, there is a fraction,  $\frac{1}{3}$ , in front of the letter.
- Guide them to see that after substitution, the unknown number will be multiplied by  $\frac{1}{3}$ , which is the same as dividing the number by 3.
- Hence, in order to get a whole number answer, our guesses for the value of  $n$  should be a multiple of 3.
- Lead students to start their guess by using  $n = 30$ .
- Get a student to substitute 30 for  $n$  in the expression ' $\frac{1}{3}n - 9$ ' and work out the answer on the board.

- Write the following on the board:  

$$'n = 30: \frac{1}{3}n - 9 = \frac{1}{3} \times 30 - 9'$$

$$= 10 - 9$$

$$= 1$$
- Have students see that when  $n = 30$ , we get an answer of 1.
- Point out to them that we want to get an answer of 0. So, the guess of  $n = 30$  is incorrect.
- Guide students in improving on their next guess. Have them see that since 1 is greater than 0, the value of  $n$  should be smaller than 30. This will enable us to get a value that is smaller than 1 when we substitute the number for  $n$  in the expression.
- Have students also see that since 1 is close to 0, the value of  $n$  must be close to 30.
- Get students to try  $n = 27$ , which is a multiple of 3 that is smaller than 30 but close to 30.
- Have another student substitute 27 for  $n$  in the expression and work out the answer on the board.
- Write the following on the board:  

$$'n = 27: \frac{1}{3} \times 27 - 9 = 9 - 9'$$

$$= 0$$
- Have students see that when  $n = 27$ , we get an answer of 0. This is the same as the value on the right side of the equal sign. So, the guess of  $n = 27$  is correct.
- Conclude that  $n = 27$  is a solution of the equation  $\frac{1}{3}n - 9 = 0$ .

#### ❖ Blended Learning Program ❖

##### From PR1ME Mathematics Interactive Edition:

###### Let's Do (CB p. 11)

Assign the tasks to students as classwork for formative assessment. Use the corresponding lesson notes to identify the objectives of each task and address remediation needs.

###### Exercise 2 (PB pp. 8–11)

Assign the tasks to students as classwork for further formative assessment. Use the corresponding lesson notes to identify the objectives of each task and address remediation needs.

##### From PR1ME Mathematics Coursework Book:

###### Coursework Book Practice 2 (CWB pp. 5–8)

Assign all tasks to students as homework. Use the following notes to identify the skills needed for each task and address remediation needs.

## Practice 2 (CWB pp. 5–8)

### Class practice (For Print-based Program):

Task 1 requires students to solve a simple algebraic equation using the guess and check method. They are required to recognize that since the first guess of  $q = 15$

arrives at an answer greater than the one given in the question, their next reasonable guess has to be a number smaller than 15.

Task 3 requires students to solve an algebraic equation that has a number in front of the letter using the guess and check method. They are required to recognize that since the first guess of  $y = 5$  arrives at an answer smaller than the one given in the question, their next reasonable guess has to be a number greater than 5.

Task 5 requires students to find out if  $n = 6$  is a solution of the algebraic equation  $\frac{1}{6}n + 28 = 32$ . They are expected to do so using the guess and check method.

#### Remediation

Task 1: Reteach solving a simple algebraic equation using the guess and check method. Then, go through Task 1. Guide students to see that the first guess of  $q = 15$  gives an answer of 24, which is greater than 20. Highlight that in order to get an answer smaller than 24, the next guess for the value of  $q$  should be smaller than 15. Guide them to try  $q = 11$  as their next guess and see that it gives an answer of 20. Conclude that  $q = 11$  is a solution of  $q + 9 = 20$ .

Task 3: Reteach solving an algebraic equation that has a number in front of the letter using the guess and check method. Then, go through Task 3. Guide students to see that the first guess of  $y = 5$  gives an answer of 3, which is smaller than 13. Highlight that in order to get an answer greater than 3, the next guess for the value of  $y$  should be greater than 5. Guide them to try  $y = 7$  as their next guess and see that it gives an answer of 13. Conclude that  $y = 7$  is a solution of  $5y - 22 = 13$ .

Task 5: Reteach solving an algebraic equation that involves a fraction using the guess and check method. Then, go through Task 5. Guide students to see that when  $n = 6$ , the answer is 29. Highlight that 29 is smaller than 32. Therefore,  $n = 6$  is not a solution of  $\frac{1}{6}n + 28 = 32$ .

#### Teaching tips

##### Tasks 1, 3 and 5

- When reteaching, follow the same procedure as the examples in Learn (CWB pp. 3–4). Draw a number line if necessary to help students see how far or close their guesses are to the solution.
- Emphasize to students that they should use logical deductions about their guesses. They should not be repeatedly choosing random numbers until they stumble upon the correct answer, nor should they try every number they can think of.
- Point out to students that they can improve on their next guess with what they have learned from the incorrect guess.
- If their guess gives an answer that is greater than the value on the right side of the equation, their next guess should be smaller than their current guess.

- If their guess gives an answer that is smaller than the value on the right side of the equation, their next guess should be greater than their current guess.

### Independent practice (For Print-based Program):

Task 2 requires students to solve simple algebraic equations using the guess and check method.

In Tasks 2(a) and 2(c), students are expected to solve each equation using addition.

In Task 2(b), students are expected to solve the equation using subtraction.

Task 4 requires students to solve algebraic equations that have a number in front of the letters using the guess and check method.

In Tasks 4(a) and 4(c), students are expected to solve each equation using multiplication and addition.

In Task 4(b), students are expected to solve the equation using multiplication and subtraction.

Task 6 requires students to solve algebraic equations that involve fractions using the guess and check method.

In Task 6(a), students are expected to solve the equation using multiplication and subtraction.

In Tasks 6(b) and 6(c), students are expected to solve each equation using multiplication and addition.

Task 7 requires students to find out if  $k = 11$  is a solution of the algebraic equation  $k + 25 = 36$ . They are expected to do so using the guess and check method.

Task 8 requires students to find out if  $j = 7$  is a solution of the algebraic equation  $8j - 11 = 29$  using the guess and check method.

Task 9 requires students to find out if  $w = 9$  is a solution of the algebraic equation  $\frac{1}{3}w + 12 = 15$ . They are expected to do so using the guess and check method.

For answers, go to CW Manual p. 96.

### ❖ Blended Learning Program ❖

#### From PRIME Mathematics Interactive Edition:

*Let's Learn (CB pp. 12–14)*

*Go through the teaching examples with students for concept development. Use the detailed lesson plan given in the corresponding lesson notes to carry out the teaching.*

### Learn

#### Using the balance method to solve algebraic equations (CWB pp. 9–11)

#### Learning Outcome:

- Use the balance method to solve an algebraic equation

#### Materials:

- 1 copy of Think About It Worksheet (WS1.2) per group
- 1 pan balance
- 20 connecting cubes of the same mass
- 3 lightweight opaque plastic bags

#### (a)

*Stage: Concrete Experience*

*In this example, students will learn to solve a simple algebraic equation using the balance method. Begin by using connecting cubes to demonstrate the balance method used to solve  $x + 6 = 10$ . This allows students to have a concrete experience of the balancing of the scale that takes place when they are solving an equation that involves one operation.*

- Set up the pan balance as shown on CWB p. 9.
- Use a bag of 4 connecting cubes to represent  $x$ . Put the bag of 4 connecting cubes on the left pan facing the students.
- Put another block of 6 connecting cubes on the left pan and a block of 10 connecting cubes on the right pan facing the students.
- Have students see that the scale is balanced.
- Explain to them that we want to find how many cubes there are in the bag.
- Guide them to see that we can do so by first removing 6 cubes from the left pan.
- Remove 6 connecting cubes from the left pan, and have students see that now the scale does not stay balanced.
- Point out to them that in order for the scale to stay balanced, we need to remove some cubes from the right pan.
- Remove a connecting cube one at a time from the right pan, until the scale is balanced.
- Have students see that 6 cubes are removed and there are 4 connecting cubes left on the right pan.
- Next, have students first recall that in an equation, the values on the left side and the right side of the equal sign are the same.
- Then, guide them to see that since the bag of cubes on the left pan balances the remaining 4 connecting cubes on the right pan, this means that the number of cubes in the bag must be the same as the number of cubes on the right pan.
- Take out the 4 cubes from the bag and reveal them to the students to show that this is true.
- Have students conclude that there are 4 cubes in the bag.